

Holiday Homework

Class: XII (CBSE)

Subject: Mathematics

1. Show that the relation R defined by $(a, b) R (c, d)$ if $a + d = b + c$ on the set $N \times N$ is an equivalence relation.
2. State whether the function $f: N \rightarrow N$ given by $f(x) = 5x$ is injective, surjective or both.
3. Let $f: N \rightarrow N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even.} \end{cases}, \text{ for all } n \text{ for all } n \in N.$$
 Find whether the function f is bijective.
4. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, find $(f \circ g)(7)$.
5. If $f: R \rightarrow R$ defined as $f(x) = \frac{2x-7}{4}$ is an invertible function, find f^{-1} .
6. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|\}$ is an equivalence relation.
7. Show that the relation R in R defined by $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.
8. Let $A = R - \{3\}$ and $B = R - \{1\}$, Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Show that f is one-one and onto.
9. If $f: R \rightarrow R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, find the value of $f \circ f(x)$.
10. Consider $f: R \rightarrow R$ given by $f(x) = 4x + 3$, show that f is invertible. Find the inverse of f .
11. Let $f(x) = [x]$ and $g(x) = x$, find $g \circ f(-5/3) - f \circ g(-5/3)$.
12. Consider $f: R^+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with

$$f^{-1}(y) = \left(\frac{\sqrt{y-6} - 1}{3} \right).$$
13. Let $*$ be a binary operation on the set of all non-zero real numbers, given by $a * b = \frac{ab}{5}$ for all $a, b \in R - \{0\}$. Find the value of x , given that $2 * (x * 5) = 10$.
14. Consider the binary operation $*$: $R \times R \rightarrow R$ and \circ : $R \times R \rightarrow R$ defined as $a * b = |a - b|$ and $a \circ b = a$ for all $a, b \in R$. Show that $*$ is commutative but not associative, \circ is associative but not commutative.
15. Consider the binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a * b = \min(a, b)$. Write the operation table of the operation $*$.