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International School

Holiday Homework

Class: XII **Subject: Mathematics**

Inverse Trigonometric functions:

1. Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$. (4 Marks)

2. Write $\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ in the simplest form (4 Marks)
   Ans: $\frac{x}{2}$

3. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x. (4 Marks)
   Ans: $x = \pm\sqrt{\frac{17}{2}}$.

4. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$. (4 Marks)

Matrices:

5. Find matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ (4 Marks)

   Ans: $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

9. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the values of 'a' and 'b'. (4 Marks)

   Ans: $a = \frac{-2}{3}$ and $b = \frac{3}{2}$

10. Express the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

11. Find the inverse of the following matrix using elementary operations:
$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$ (4 Marks)

   Ans: $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Determinants:

12. Using properties of determinants, solve for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$

Ans: $x = 0, 3a$. (4 Marks)

13. Show that the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies the equations $A^2 - 5A + 7I = O$. Hence, find A^{-1}.

Ans: $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (4 Marks)

14. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of equations : $x - y = 3$, $2x + 3y + 4z = 17$, and $y + 2z = 7$. (6 Marks)

Ans: $AB = 6I$, $x = 2$, $y = -1$, $z = 4$.

Continuity and Differentiability:

15. Find the relationship between 'a' and 'b' so that the function 'f' defined by

$$f(x) = \begin{cases} ax + 1, & if \ x \leq 3 \\ bx + 3, & if \ x > 3 \end{cases}$$ is continuous at $x = 3$. (4 Marks)

Ans: $a - b = \frac{2}{3}$