

Holiday Homework

Class: XII

Subject: Mathematics

Inverse Trigonometric functions :

1. Prove that $2 \sin^{-1} \theta = \sin^{-1} [2\theta\sqrt{1-\theta^2}]$. (4 Marks)

2. Write $\tan^{-1} \sqrt{\frac{1-\cos 3x}{1+\cos 3x}}$ in the simplest form (4 Marks)

Ans : $\frac{3x}{2}$.

3. Solve $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$. (4 Marks)

Ans: $x = \frac{\sqrt{3}}{2\sqrt{7}}$.

4. Show that $\sin^{-1} \frac{3}{5} - \sin^{-1} \frac{8}{17} = \cos^{-1} \frac{84}{85}$. (4 Marks)

5. Show that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$. (4 Marks)

6. Prove that $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$. (4 Marks)

7. Prove that $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x, -\frac{1}{\sqrt{2}} \leq x \leq 1$. (4 Marks)

Matrices :

8. If $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 5 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{bmatrix}$, then find B. (4 Marks)

Ans: $B = \frac{1}{3} \begin{bmatrix} -2 & 1 & 19 \\ -5 & 8 & 11 \end{bmatrix}$.

9. Find x, if $[x \ 1] \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$. (4 Marks)

Ans: $x = 1 \pm \sqrt{10}$.

10. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $B = [-2 \ -1 \ -4]$, verify that $(AB)' = B'A'$. (4 Marks)

11. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Determinants :

12. Compute A^{-1} for the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ and show that $A^{-1} = \frac{1}{19} A$. (4 Marks)

13. Using the properties of determinants, prove that

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2. \quad (4 \text{ Marks})$$

14. Using the properties of determinants, prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$. (4 Marks)

Note : Learn the Differentiation formulae which I sent to your tabs as Audio Video Lecturer.